MATH TO MAKE YOU GO HMMM

PACK 1

Attached are a number of mathematical concepts that are, well, different. They involve number theory or geometrical reasoning but are not taught at the elementary level, but many students can begin accessing the concepts or tasks. This doesn't mean that students will "master" them as the full algebra and geometry behind the tasks are at a high school level or beyond. So, please treat them as puzzles and if your student tires of them or becomes frustrated, the Discovery teachers understand and it is OK to stop the math.

PUZZLE 1: SIEVE OF ERATOSTHENES: Greek mathematician Eratosthenes discovered an algorthim (a rule or process) for finding prime numbers.

PUZZLE 2: TRIANGULAR NUMBERS: Most of us learn square number, but there are a whole additional set of number types, such as Triangular number. In the puzzle, most students will understand the concept but not be able to calculate the pattern out to 30 or 100 or *n* (the algebraic general rule). This is A-OK.

PUZZLE 3: Toothpick Geometry: Can you make one toothpick shape into another shape with only a few moves???

PRIME NUMBERS AND THE SIEVE OF ERATOSTENES

What is a prime number?

A prime number is a number (greater than 1) that is only divisible by 1 and itself.

What are composite numbers?

They are numbers (greater than 1) that are not prime. They are divisible by 1, by itself and by other numbers.

Let's look at an example of a prime and composite number

The number 11 can be written as 1 x 11, but cannot be written as a multiplication problem of any other natural numbers. It only has the factors 1 and 11.
Therefore, **11 is a prime number.**

The number 12 can be written as the 1 x 12, 3 x 4, and 2 x 6. Since 12 is divisible by numbers other than 1 and itself, **12 is a composite number.**

The Sieve of Eratostenes – A way to find prime numbers:

HOW TO DO

Step 1) Circle the first number (which is number 2). This number is prime.

Step 2) Cross out all the multiples of the number on your list. You would cross out the numbers 4, 6, 8, 10, 12, ...

These numbers have been 'sieved' out.

Step 3) Circle the next number on your list that has not been sieved out. (Number 3). This number is prime.

Step 4) Cross out all the multiples of 3 on your list.

Repeat this process until all the numbers on your list are either circled or crossed out. The numbers circled are *prime*. The numbers crossed out are *composite* (or non-primes).

Name Date

SIEVE OF ERATOSTHENES TO 200

- Start at number 2. Put a circle it. Cross out all the multiples of that number.
- Move on to the next number you have not crossed out (3). Put a circle round it. Cross out all the multiples of that number. Repeat

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

How many prime numbers are there between 1 and 200?

Name Date

SIEVE OF ERATOSTHENES TO 200 ANSWERS

- The numbers highlighted are all primes.

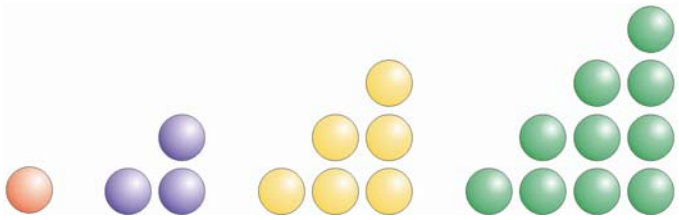
	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

How many prime numbers are there between 1 and 200? 46

Triangular Numbers

NAME _____

Triangular numbers can be represented by dots arranged in a triangle.

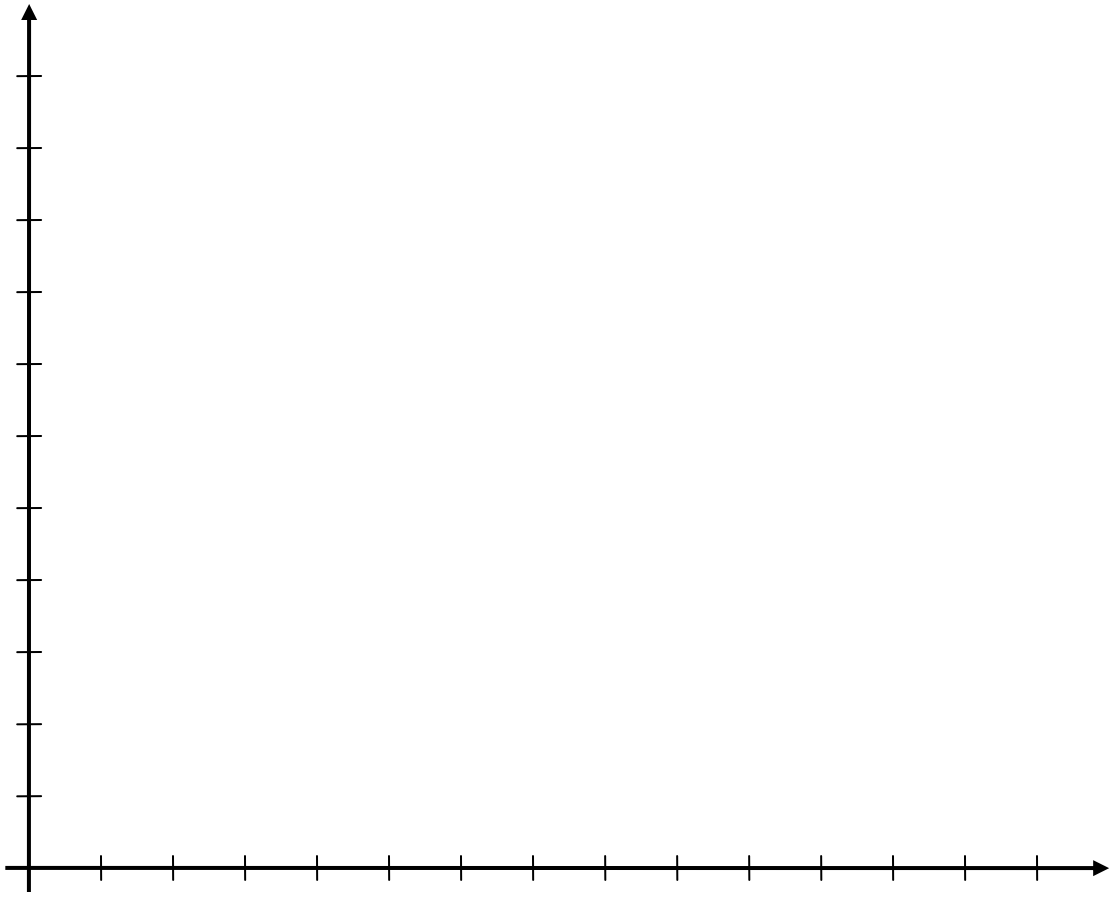


1 **3** **6** **10**

The first triangular number is 1, the second is 3, the third is 6, and so on. Extend the pattern to complete the table below. As necessary, draw additional figures to help you.

TRIANGULAR NUMBER	DOTS
1	1
2	3
3	6
4	10
5	
6	
10	
30	
100	
n	

Using the numbers from the table on the previous page, graph the results below. Plot the figure number along the horizontal axis, and plot the number of dots along the vertical axis. (Be sure to label both axes.)



Toothpick Geometry

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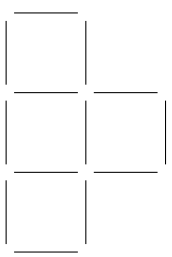
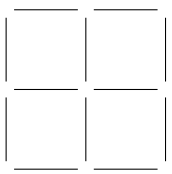
The goal of this activity is to develop geometric visualization skills and proof techniques. You will be given a toothpick configuration and will have to remove or rearrange a specified number of toothpicks to create a new configuration with certain attributes. As you master each problem, you will see how solving it can be useful for solving the harder problems. It should be noted that several of these problems have multiple solutions. Pay attention to the instructions carefully. Good luck!

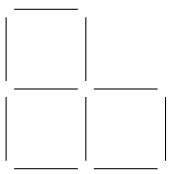
1. Remove one toothpick to leave only three squares.



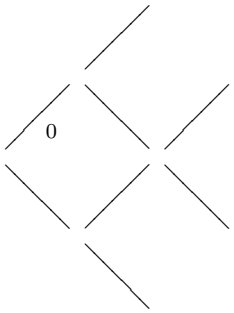
2. Remove two toothpicks to leave only two squares.



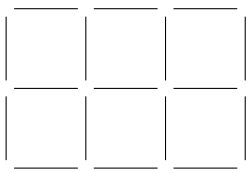
3. Remove two toothpicks to leave only two squares.



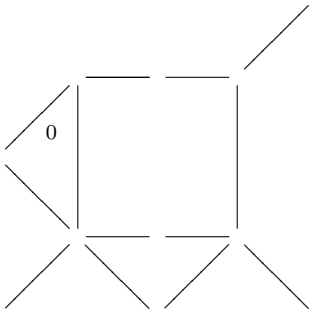
4. Make the fish swim the opposite way by moving three toothpicks.



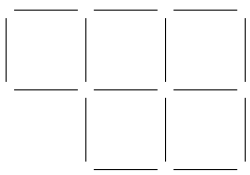
5. Remove six toothpicks to leave only two squares.



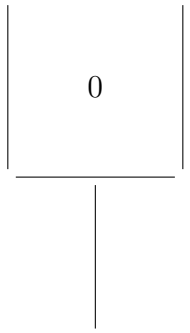
6. Move two toothpicks to make the pig go the opposite way (he will be running scared!).



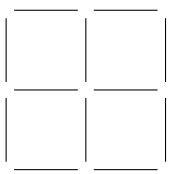
7. Remove three toothpicks to leave only three squares.



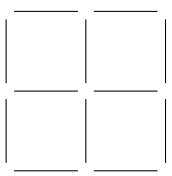
8. Move two toothpicks to get the ball out from between the goalposts.



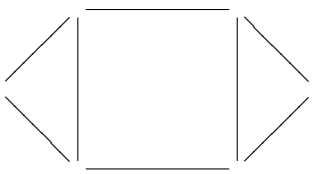
9. Move four toothpicks to make only three squares.



10. Move four toothpicks to make only four triangles and only two squares.

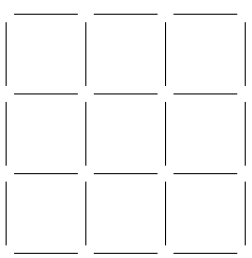


11. Move two toothpicks to make only five triangles and only one square.

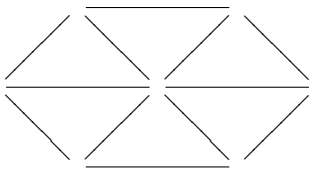


From here on, they get harder!!!

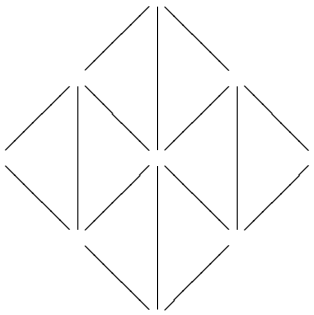
12. Remove eight toothpicks to leave only three squares.



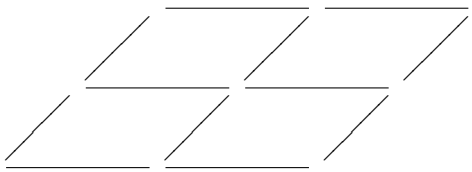
13. Move four toothpicks to leave only three equilateral triangles (the diagram is not drawn to scale).



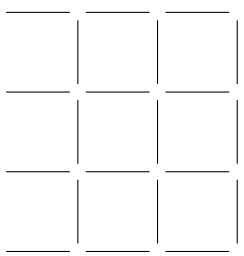
14. Remove six toothpicks to leave only four triangles.



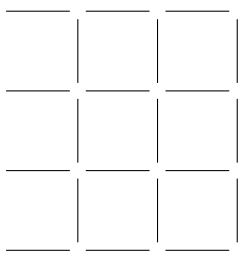
15. Move two toothpicks to make only four triangles.



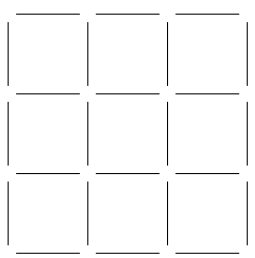
16. Move six toothpicks to make only twelve triangles and only seven squares.



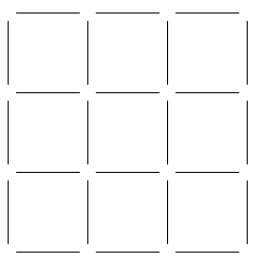
17. Move six toothpicks to make only seventeen triangles and only seven squares.



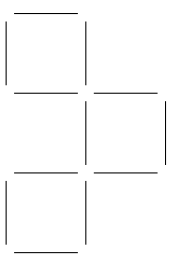
18. Move four toothpicks to make only eight triangles and only seven squares.

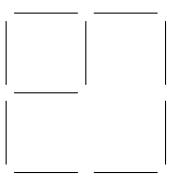


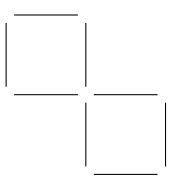
19. Move four toothpicks to make only nine triangles and only seven squares.

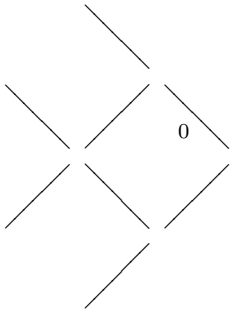


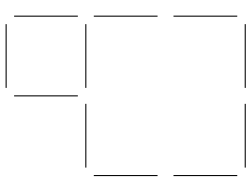
Solutions

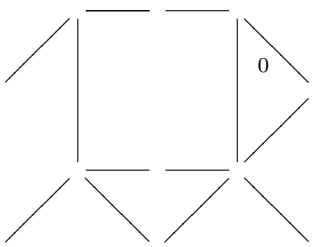
1. 

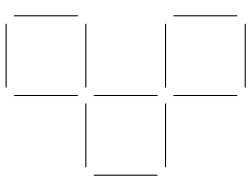
2. 

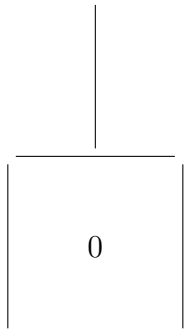
3. 

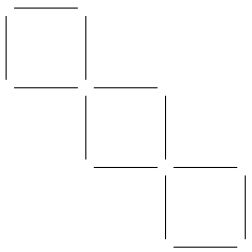
4. 

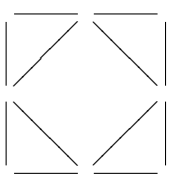
5. 

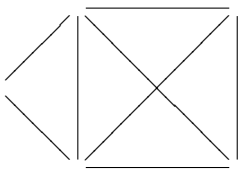
6. 

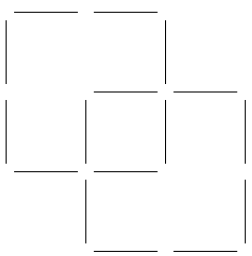
7. 

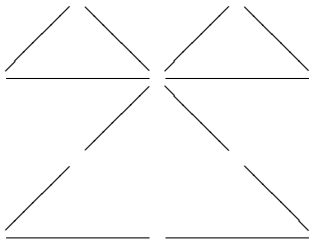
8. 

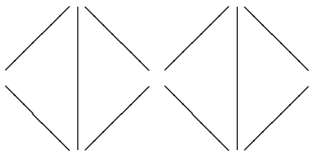
9. 

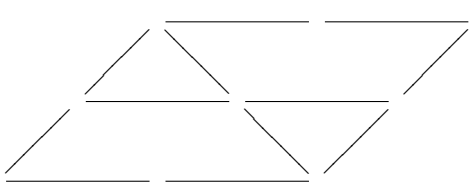
10. 

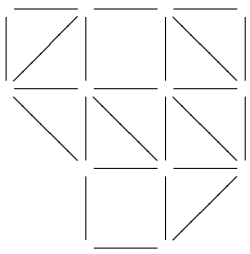
11. 

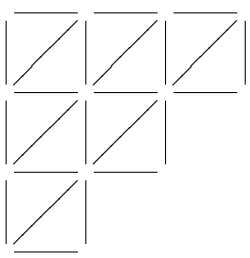
12. 

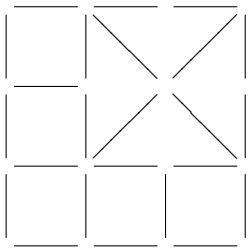
13. 

14. 

15. 

16. 

17. 

18. 

19.